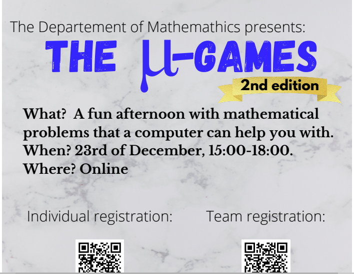
μ - Games

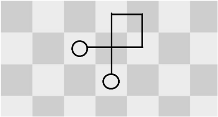
Mathematics Utrecht

Utrecht University

23 December 2021



A Super Knight can travel to all the squares of the same color:



To determine what color a knight/king is one, calculate if $x_i + y_i$ is odd or even.

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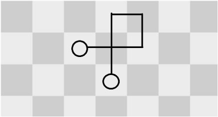

To determine what color a knight/king is one, calculate if $x_i + y_i$ is odd or even.

Looking from biggest to smallest:
If one fits, you want to take it with you because of the superincreasing property

- $N = 1000000$.
- $I = \{a, b, c, d, e, f, g, h, i, j\}$.
- Note that $9! < N < 10!$.

To determine the first letter of the permutation, we calculate

$(1000000 - 1) \mod (9!) = 274239, (999999 - (999999 \mod (9!))/9! = 2$
Number remaining: 274239, Letters remaining: $\{a, b, d, e, f, g, h, i, j\}$

$274239 \mod (8!) = 32319, (274239 - (274239 \mod (8!))/8! = 6$
Number remaining: 32319, indices remaining: $\{a, b, d, e, f, g, i, j\}$

$32319 \mod (7!) = 2079, (32319 - (32319 \mod (7!))/7! = 6$

Number remaining: 2079, indices remaining: $\{a, b, d, e, f, g, j\}\vdots$

$\vdots$

In the end, we have the permutation $chidjbfega$.

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Using the Euler discretization we have

$$y_{n+1} = y_n + h \cdot (ky_n) = (1 + hk)y_n.$$

Hence, we have $y_n = (1 + hk)^n y_0 = (1 + hk)$. This converges exactly on the set

$$\{h \in \mathbb{R} \geq 0 \ : \ |1 + hk| < 1\}.$$

So to determine if $\lim_{n\to\infty} y_n = 0$, we calculate $|1 + hk|$ and see if it is smaller than 1.

Using the Trapezoid discretization we have

$$y_{n+1} = y_n + \frac{1}{2}h(ky_n + ky_{n+1})$$

Solving for y_{n+1} gives

$$y_{n+1} = \frac{1 + \frac{1}{2}hk}{1 - \frac{1}{2}hk} y_n.$$

This converges exactly on the set

$$\{h \in \mathbb{R}\geq 0 \ : \left|\frac{1 + \frac{1}{2}hk}{1 - \frac{1}{2}hk}\right| < 1\}.$$

So to determine if $\lim_{n\to\infty} y_n = 0$, we calculate $|\frac{1+\frac{1}{2}hk}{1-\frac{1}{2}hk}|$ and see if it is smaller than 1.

With the algorithm, we can find x such that:

$$rx + qy = \gcd(r, q) = 1.$$

So

$$rx \equiv 1 \mod q$$

.

We see

$$\sum_i m_i b_i = \left(\sum_i m_i w_i r\right) + q \cdot L_1$$

We can multiply this by x:

$$\left(\sum_i m_i w_i r x\right) + q \cdot L_2 = \left(\sum_i m_i w_i\right) + q \cdot L_3$$

Now since $q > \sum_i w_i$ and the $m_i \in \{0, 1\}$, we can do modulo q:

$$\left(\sum_i m_i w_i\right) + q \cdot L_3 \equiv \sum_i m_i w_i$$

Now since we know the w_i, this is the Lunchbox problem from earlier.

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$$f(x_1, x_2, \ldots, x_n) = \prod_{j=1}^{k} (\alpha_{j,1} x_{i_{j,1}} + \alpha_{j,2} x_{i_{j,2}} - \min(\alpha_{j,1}, 0) - \min(\alpha_{j,2}, 0))$$

$$\alpha_{j,1}, \alpha_{j,2} = -1, 1$$

- we are only interested in the value of f at $x_1, \ldots, x_n = 0, 1$.
- Note that each term evaluates either to 0 or 1, and when $\alpha = -1$, you actually have 1-x in the corresponding term.
- *e.g.* $(x_1 + x_2)(x_3 + x_4)$ becomes $(x_1 \vee x_2) \wedge (x_3 \vee x_4)$
- each term can be split into two implications: e.g. $x_1 \vee x_2$ makes $(\sim x_1 \rightarrow x_2) \wedge (\sim x_2 \rightarrow x_1)$
- these implications can make chains, e.g. $x_1 \rightarrow x_2 \rightarrow \sim x_3$ and if such a chain contains a loop $x \rightarrow \sim x$ and $\sim x \rightarrow x$ we have a contradiction, hence $f = 0$ for all inputs.

Example with no loops

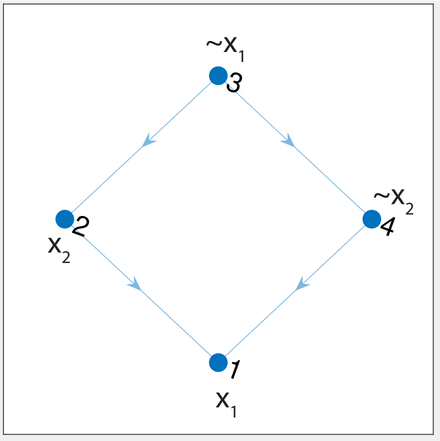
2
2
1 1 1 2
1 -1 1 2

$$f=(x_1+x_2)(x_1+1-x_2)$$

$$(x_1 \text{ or } x_2) \text{ and } (x_1 \text{or} \sim x_2)$$

$$\sim x_1 \to x_2 \qquad \sim x_1 \to \sim x_2$$

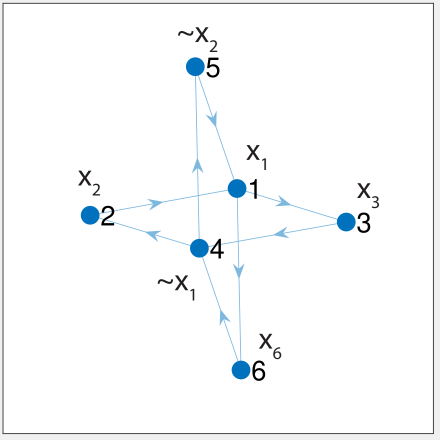
$$\sim x_2 \to x_1 \qquad x_2 \to x_1$$



$$Contra\ example\ x_1=1,\ x_2=1,\ f=1$$

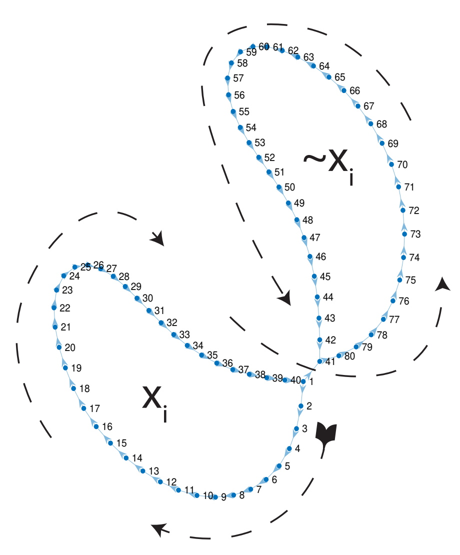
Example with loops

3
4
1 1 1 2
1 -1 1 2
-1 1 1 3
-1 -1 1 3

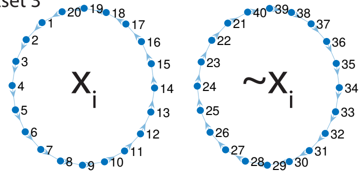


More examples

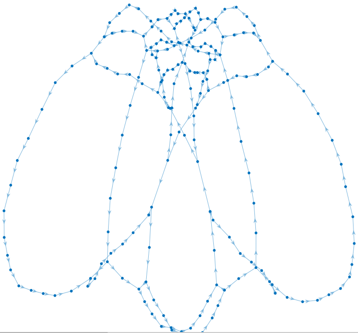
Testset 4



Testset 3



Testset 9



Testset 5: